Approximate Continuation of Harmonic Functions in Geodesy

A Weighted Least-Squares Approach Based on Splines with Extension to the Multiscale Adaptive Case

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Abstract

The reconstruction of the earth's potential field from geometric and gravimetric data is an ill-posed data fitting problem with tens of thousands of parameters. One employs its essential property stating that the potential field is harmonic, i.e., it fulfills the Laplace equation. In practice, the representation and determination of the gravity field includes the continuation of gravity measurements together with the treatment of noise and outliers. The representation of fields in terms of spherical harmonics is computationally very expensive and confined to spherical domains. Here, we investigate formulations based on regularized least-squares functionals and localized representations based on multiscale finite elements and wavelets. We construct continuations of potential fields using only local information by a data fitting ansatz with respect to the boundary conditions corroborated with a simultaneous regularization enforcing the harmonicity over the interior of the domain. We employ generalized multiscale finite element and wavelet approaches to higher orders. For the adaptive version of our method we employ a representation in terms of a hierarchical B-splines construction. The approach works with an iterative, coarse-to-fine algorithm and differently designed refinement strategies. We also investigate the determination of the weight parameter for the regularization and additional parameters of the algorithm, like thresholding parameters and stopping criteria.